

Temperature and Density Effects on the Nucleon Mass Splitting*

H. R. Christiansen, L. N. Epele, H. Fanchiotti

and C. A. García Canal

Departamento de Física, Universidad Nacional de La Plata

C.C. 67, (1900) La Plata, Argentina

Abstract

The finite temperature and finite density dependence of the neutron-proton mass difference is analysed in a purely hadronic framework where the $\rho - \omega$ mixing is crucial for this isospin symmetry breakdown. The problem is handled within Thermo Field Dynamics. The present results, consistent with partial chiral and charge symmetry restoration, improve the experimental data fit for the energy difference between mirror nuclei.

PACS numbers: 14.20.Dh 13.75.Cs 21.65.+f 11.10.Wx

Typeset using REVTeX

*The authors are partially supported by CONICET, Argentina.

I. INTRODUCTION

Symmetry restoration in spontaneously broken gauge theories has been one of the first applications of field theory at finite temperature [1]. In the Standard Model, such a phase transition is supposed to have taken place in the early Universe when the temperature was of the order of a few hundred GeV. Different types of phase transitions are also expected to occur at much lower temperatures and with hadronic non zero densities. In connection with it, the broken chiral symmetry has been predicted to be restored at about the same temperature at which the deconfinement sets in. This temperature is in the range of 150 to 200 MeV as it is indicated from lattice calculations [2] and phenomenological models based on effective Lagrangians [3]. High density conditions are supposed to have similar consequences on the hadronic matter.

In the nuclear scale there are also indications of partial deconfinement in order to explain the EMC effect [4]. It has been argued that nucleons in nuclei occupy a larger volume than nucleons in vacuum [5], a phenomenon that could be understood as a nucleon swelling [6]. It has been also analysed in connection with the Nolen-Schiffer Anomaly (NSA) [7] for different models of the nucleon [8]- [10].

In recent years, the possibility of producing quark-gluon plasma by means of very energetic nucleus-nucleus collisions [11] opened a rather important program of investigation of matter under extreme conditions that could shed light on the fundamental problems mentioned above.

In the present work we analyse the behavior of the neutron-proton mass difference with temperature and density, within meson theory. Our results are consistent with both partial chiral and charge symmetry restoration, with increasing temperature and density. In particular, the density effects are in order to clear away the Nolen-Schiffer Anomaly.

Although a full description of the neutron-proton mass difference calls for a non perturbative approach, it is still lacking. Nevertheless, one may gain a good understanding of the problem, relying on the perturbative methods used in many body nuclear physics. There,

relativistic perturbation theory (RPT) is mainly used for the analysis of hadron interactions [12].

In a previous article [13], we have obtained a good outcome for the n-p mass splitting using RPT at the hadronic level. In that work we have shown that the role of the rho-omega mixing interaction is crucial in the understanding of the n-p mass difference, in a *hadronic context* (see also [8]). We have concluded that the mixing of the vector mesons coming from the u-d mass difference, has important consequences on the nucleon self-energy and is the main non-electromagnetic charge symmetry breaking (CSB) contribution to be considered. This *effective* interaction has been thoroughly investigated since 1960 [14], particularly within the tadpole picture [15]- [17]. It has been generally claimed that the main source for that vertex has a non-electromagnetic origin related to the quark mass difference [17,18]. The connection between the mixing lagrangian of eq.(1) and the microscopic quark level has been recently explored within different models: QCD sum rules [19], Coleman-Glashow tadpoles [20], constituent quark model [18] and the Nambu–Jona-Lasinio model [21].

Notice that $\Delta M_{n-p}^{exp} = 1.29$ MeV while $\Delta M_{n-p}^{em} = -0.66$ to -0.76 MeV [22,23], implying a 2 MeV strong contribution. Within the scheme of ref. [13] this quantity can be obtained from the nucleon self-energy given in eq.(2), using the experimental values of couplings and meson masses in the literature [15]- [18], [24].

II. THE MODEL

In the present work we obtain the finite temperature and density (FTD) dependence of the nucleon mass splitting using the same framework as in ref. [13] where we have considered the nucleon and the vector mesons as the fundamental dynamical degrees of freedom. In this context, the calculation starts from the following Hamiltonian [15]

$$\begin{aligned} \mathcal{H}_I = & \frac{1}{2}g_\rho \bar{N}(p')(\gamma^\mu + \frac{k^V}{2M}i\sigma^{\mu\nu}q_\nu)\vec{\tau}\vec{\rho}_\mu(q)N(p) \\ & + \frac{1}{2}g_\omega \bar{N}(p')\gamma^\mu\omega_\mu(q)N(p) + \lambda\rho_\mu^0(q)\omega^\mu(q) \end{aligned} \quad (1)$$

which corresponds to the standard minimal formulation of the interaction under consideration. Here g_ρ and g_ω are the vector meson coupling constants, k^V is the isovector anomalous magnetic moment, $M_n = M_p = M$, m_ρ and m_ω are the nucleon and mesons masses respectively and λ is the mixing matrix element. The lowest order self-energy correction to the nucleon mass coming from \mathcal{H}_I , which explicitly breaks the charge symmetry is

$$\begin{aligned}
-i\Sigma^F(p) = & \int \frac{d^4q}{(2\pi)^4} \\
& \left\{ \frac{g_\rho}{2}(\gamma^\mu + \frac{k^V}{2M}i\sigma^{\mu\nu}q_\nu)\tau_3 S(p-q)\frac{g_\omega}{2}\gamma_\mu\lambda D_\rho(q)D_\omega(q) + \right. \\
& \left. \frac{g_\omega}{2}\gamma^\mu S(p-q)\frac{g_\rho}{2}(\gamma_\mu - \frac{k^V}{2M}i\sigma_{\mu\nu}q^\nu)\tau_3\lambda D_\omega(q)D_\rho(q) \right\} \quad (2)
\end{aligned}$$

where

$$S(p-q) = \frac{(\not{p}-\not{q}+M)}{(p-q)^2-M^2} \quad \text{and} \quad D_v(q) = 1/(q^2 - m_v^2).$$

We will also include form factors at the meson-nucleon vertices to take into account the effect of the nucleon structure in a phenomenological way, thus giving to the model a wider range of application in its momentum dependence [13]

$$F_v(q) = \frac{\Lambda_v^2 - m_v^2}{\Lambda_v^2 - q^2} \quad \text{where } v = \rho, \omega$$

A natural framework for the study of matter under FTD conditions is the so-called Thermo Field Dynamics (TFD) [25]. TFD is a real time formalism of the statistical field theory, very powerful for describing non-isolated many body systems. It is a canonical field theory formulation in which the Hilbert space is doubled and each field operator has two independent components belonging to the thermal doublet. Correspondingly, the Green's functions, self-energies etc., are expressed by the thermal matrices. By virtue of this extension of the Hilbert space, the pathologies of the pioneer formulations of the real time formalism approach are avoided [26]. Moreover because the Gell-Mann–Low formula and the Wick's theorem for the perturbation expansion are available in TFD, the usual perturbation theory at zero temperature and density can be easily extended to FTD. This formalism

is also particularly useful to perform both high and low temperature expansions, a feature much less accessible in the imaginary time formalism [27]. Consequently, perturbation theory is at hand using the Feynman diagrams technique proper of RPT. Moreover, the TFD free propagators can be explicitly separated into two parts: one term being the usual one and the other part depending on temperature and density. For the sake of brevity we will not discuss the derivation of the corresponding thermal matrices for the modified propagators to be used in the present calculation [28]. For fermions one gets

$$\begin{aligned}
S_{\alpha\beta}^{ab}(k) &= S_{\alpha\beta}^{ab,F}(k) + S_{\alpha\beta}^{ab,z}(k) \\
&= (\not{k} + M)_{\alpha\beta} \left(\begin{pmatrix} \frac{1}{k^2 - M^2 + i\epsilon} & 0 \\ 0 & \frac{1}{k^2 - M^2 - i\epsilon} \end{pmatrix} \right. \\
&\quad \left. + 2\pi i \delta(k^2 - M^2) \begin{pmatrix} \sin^2(\theta_{k_0}) & \frac{1}{2} \sin(2\theta_{k_0}) \\ \frac{1}{2} \sin(2\theta_{k_0}) & -\sin^2(\theta_{k_0}) \end{pmatrix} \right)
\end{aligned} \tag{3}$$

with

$$\begin{aligned}
\cos(\theta_{k_0}) &= \frac{\Theta(k_0)}{(1 + e^{-z})^{1/2}} + \frac{\Theta(-k_0)}{(1 + e^z)^{1/2}} \\
\sin(\theta_{k_0}) &= \frac{e^{-z/2}\Theta(k_0)}{(1 + e^{-z})^{1/2}} - \frac{e^{z/2}\Theta(-k_0)}{(1 + e^z)^{1/2}}
\end{aligned} \tag{4}$$

where $z = (k_0 - \mu)/k_B T$, μ is the relativistic chemical potential and $\Theta(k_0)$ is the step function. Similarly for the meson propagator one obtains

$$\begin{aligned}
D^{ab}(k) &= D^{ab,F}(k) + D^{ab,z}(k) \\
&= \left(\begin{pmatrix} \frac{1}{k^2 - m_v^2 + i\epsilon} & 0 \\ 0 & \frac{-1}{k^2 - m_v^2 - i\epsilon} \end{pmatrix} \right. \\
&\quad \left. - 2\pi i \delta(k^2 - m_v^2) \begin{pmatrix} \sinh^2(\phi_{k_0}) & \frac{1}{2} \sinh(2\phi_{k_0}) \\ \frac{1}{2} \sinh(2\phi_{k_0}) & -\sinh^2(\phi_{k_0}) \end{pmatrix} \right)
\end{aligned} \tag{5}$$

where the trigonometric functions for the particle-antiparticle distributions have to be replaced by the corresponding hyperbolic ones [28]. The $(a, b) = (1, 1)$ elements of the thermal matrices are the standard Feynman physical propagators while the $(a, b) = (2, 2)$ are the so-called thermal ghost partenaires and the off-diagonal terms are mixed in nature. TFD establishes that inner vertices of a diagram can be of either type, physical or ghost while the external ones ought to be physical. In view of this, and up to second order perturbation, only the mixing vertex λ involve both kind of terms. However, for the ghost type, the product of meson propagators will produce a vanishing contribution to the loop. Consequently, the full FTD expression for the second order self energy results when S^{11} replaces S and D_v^{11} replaces D_v in eq.(2). Now we can separate Σ as

$$\begin{aligned}\Sigma = \Sigma^F + loop(S^{11,z} D_\rho^{11,F} D_\omega^{11,F}) \\ + loop(S^{11,F} D_\rho^{11,z} D_\omega^{11,F}) + loop(S^{11,F} D_\rho^{11,F} D_\omega^{11,z})\end{aligned}\quad (6)$$

where the first term, $\Sigma^F = loop(S^F D_\rho^F D_\omega^F)$, represents the usual zero temperature and density Feynman contribution calculated in Ref. [13], and the others are finite T - δ corrections. On the other hand, for nucleons on-shell $\delta M_p = \bar{u}_p(p) \Sigma u_p(p) = -\bar{u}_n(p) \Sigma u_n(p)$, with $p^2 = M^2$, then, performing the q_0 integration, we obtain for the second term in eq.(6) (see fig.(1))

$$\begin{aligned}-i\delta^c M_p = -i \frac{\pi^2 g_\rho g_\omega \lambda}{(2\pi M)^4} (\Lambda_\rho^2 - m_\rho^2)(\Lambda_\omega^2 - m_\omega^2) \tau_3 \int_M^\infty d\varepsilon \frac{1}{e^{\beta(\varepsilon - \mu)} + 1} \\ \frac{\sqrt{\varepsilon^2 - M^2} ((8 + 6k^V)M - (4 + 6k^V)\varepsilon)}{(\frac{2M^2 - m_\rho^2}{2M} - \varepsilon)(\frac{2M^2 - m_\omega^2}{2M} - \varepsilon)(\frac{2M^2 - \Lambda_\rho^2}{2M} - \varepsilon)(\frac{2M^2 - \Lambda_\omega^2}{2M} - \varepsilon)}\end{aligned}\quad (7)$$

where the contribution coming from antiparticles has been omitted since we will consider temperatures much below pair production, $k_B \beta^{-1} \ll M c^2$. Furthermore, as the presence of real mesons calls for extreme conditions, the in-medium corrections to the bosonic propagators included in the last two terms of eq.(6), shall be neglected.

Since our description has been given in terms of effective fields and couplings, it wouldn't make sense to explore our system in the high T - δ regime where total deconfinement could take place. Hence it is natural to consider $T \ll M$ and densities not too high. For arbitrary T and δ conditions, eq.(7) must be solved numerically, nevertheless one can find usefull expressions for some physical scenarios. To this end one has to find accordingly, aproximated formulas to the chemical potential which is defined in terms of temperature and density by

$$\delta = \frac{4}{(2\pi\hbar)^3} \int d^3k \left(\frac{1}{e^{(k_0-\mu)/k_B T} + 1} - \frac{1}{e^{(k_0+\mu)/k_B T} + 1} \right) \quad (8)$$

At finite density, the chemical potential (in units $c = \hbar = k_B = 1$) can be aproximated by

$$\mu \simeq M + T \log \frac{\delta}{4} \left(\frac{2\pi}{MT} \right)^{3/2} \quad (9)$$

provided that [30]

$$2\pi \left(\frac{\delta}{4} \right)^{2/3} \ll TM \ll M^2 \quad (10)$$

Using eq.(9) at low temperatures gives a very small value for the self-mass correction $\delta^c M_p$ (see next section). In particular, $\delta^c M_p$ vanishes as $T \rightarrow 0$.

On the other hand, in the range of the nuclear matter density, a T^2 computation of μ results in

$$\begin{aligned} \mu^2 &\simeq \mu_0^2 - \frac{\pi^2}{3} \left(1 + \frac{\mu_0^2}{\mu_0^2 - M^2} \right) T^2 \\ \mu_0^2 &= M^2 + \left(\frac{6\pi^2}{4} \delta^{2/3} \right) \end{aligned} \quad (11)$$

which is a good aproximation for

$$T^2 \ll \frac{\frac{9}{2} \delta^{4/3}}{(3\pi^2 \delta^{2/3} + M^2)} \quad (12)$$

Now, performing a standard low T expansion [29] in eq.(7), we get

$$\begin{aligned} \delta^c M_p \simeq & \frac{\pi^2 g_\rho g_\omega \lambda}{(2\pi M)^4} (\Lambda_\rho^2 - m_\rho^2) (\Lambda_\omega^2 - m_\omega^2) \left(\int_M^{\mu_0} g(\varepsilon) d\varepsilon \right. \\ & \left. + \frac{\pi^2}{6} T^2 (g'(\mu_0) - g(\mu_0) \frac{2\mu_0^2 - M^2}{(\mu_0^2 - M^2)\mu_0^2}) \right) \end{aligned} \quad (13)$$

with

$$g(\varepsilon) = \frac{\sqrt{\varepsilon^2 - M^2} ((8 + 6k^V)M - (4 + 6k^V)\varepsilon)}{\left(\frac{2M^2 - m_\rho^2}{2M} - \varepsilon\right) \left(\frac{2M^2 - m_\omega^2}{2M} - \varepsilon\right) \left(\frac{2M^2 - \Lambda_\rho^2}{2M} - \varepsilon\right) \left(\frac{2M^2 - \Lambda_\omega^2}{2M} - \varepsilon\right)} \quad (14)$$

In this context, it can be seen from eq.(13), that $\delta^c M_p$ is significant even at $T = 0$. Equations (11)-(13) are good approximations for astrophysical temperatures (a couple of MeV's) and baryon densities in the range of the nuclear matter density $\delta_0 = 0.1934 fm^{-3}$ [31]. Under these conditions, the T^2 -dependent term of eq.(13) is negligible. Hence, the main contribution comes from the first term, which only depends on δ (see next section).

Therefore, at finite density equations (9) and (11) have smooth limits for $T \rightarrow 0$, and so does eq.(7). On the other hand, at finite temperature and very low densities, one has $\mu = \delta \cdot f(\beta)$, implying that the limit $\delta \rightarrow 0$ is also smooth for the self-mass correction (7). In this case, $\delta^c M_p$ depends on $e^{-M/T}$ and is exponentially small for $T \ll M$. The analysis above shows that in the limit of vanishing chemical potential and zero temperature, $\delta^c M_p \rightarrow 0$ as expected.

III. NUMERICAL RESULTS AND FINAL CONCLUSIONS

In order to obtain numerical predictions from this approach we take experimental values for all the physical quantities appearing in our formulas: $m_\omega = 783$ MeV, $m_\rho = 770$ MeV, $M = 939$ MeV, $k^V = 3.7$. Concerning the coupling constants we work with a set $g_\rho g_\omega \lambda = 0.492$ GeV² chosen in order to saturate the 2 MeV hadronic contribution to the zero temperature and density nucleon mass difference within the model. This election is

well inside the accepted range of variation of these coupling constants found in the literature [15,16,18,24].

As it is known, eq.(9) is mostly suitable for high temperatures and low densities, nevertheless as our model is not expected to be reliable far from deconfinement ($T_D \simeq 180$ MeV), we should consider MT below T_D^2 instead of M^2 , together with conditions (10). For example, at $T \leq 15$ MeV and densities satisfying these conditions (say below $\delta_0/10$), the value of $\Delta^c M = \delta^c M_n - \delta^c M_p$ is below a couple of keV, which is negligible (recall that in the chosen units, $1fm = 197 \text{ MeV}^{-1}$). For low temperatures and baryon densities ranging from $\delta_0/2$ to $1.5\delta_0$, we have to use eqs.(11)-(13). In this case, in turn, $\Delta^c M$ goes from -0.1 to -0.25 MeV (see fig. 2).

Evidently, it is the non-trivial shape of $\mu(\delta, T)$ which is responsible for the behavior of the in-medium mass difference. As we have mentioned, the temperature contribution is negligible for astrophysical temperatures, which we are interested in.

In connection with these results on the neutron-proton mass difference as a function of the nuclear density, let us finally comment on the Nolen-Schiffer Anomaly. The NSA is a persistent problem in nuclear physics and the anomaly is the failure of theory to explain the mass differences between mirror nuclei (*i.e* nuclei with $Z = A \pm 1/2$ and $N = A \mp 1/2$), a gap amounting to a few hundred keV. The effects of nuclear structure have been widely discussed with only partial success [32]. The mass difference of mirror nuclei can be written as $M_{Z>} - M_{Z<} = \Delta E_{em} - (M_n - M_p)$ where $M_{Z>}$ is the mass of the nucleus with the larger charge, ΔE_{em} is the electromagnetic self energy difference between the nuclei and $M_n - M_p$ is the nucleon mass difference inside the nucleus.

Since ΔE_{em} has been exhaustively analysed, in recent years particular attention has been paid to the second term. A variety of models have been put forward in order to avoid this problem. Generally it is found that $M_n - M_p$ is a decreasing function of the nuclear density [8]- [10]. It means that within these models high density is also expected to produce a partial charge symmetry restoration. This is the expected behavior to deal with the anomaly.

In this way, the final results of our calculation are in the right direction to remove the

anomaly.

It should be mentioned that some recent literature has suggested that the λ (off-shell) momentum dependence could be important [35]. However, this conclusion resulted from the use of oversimplified models for the $\rho - \omega$ correlation function. In fact, it has been recently shown [19] that the effect of the different widths of the ρ and ω mesons implies that λ is almost momentum independent. This is consistent with the present understanding of the CSB phenomenology [36,37], where the $\rho - \omega$ mixing has shown to be of particular importance.

Following similar steps to include the electromagnetic interaction in the current scheme, we have found that the nuclear medium has a relatively strong effect on the electromagnetic self-energy ΔM_{n-p}^γ in the opposite direction, amounting to about 15% of its effect on $\Delta M_{n-p}^{\rho\omega}$. This is in good agreement with the results of Ref. [8] which have been derived making a quite different treatment of the problem, in order to include the external conditions by means of Skyrme type models.

In table 1 we show the values of NSA reported in the literature. From this table it emerges (a rough) accordance between the predictions of the different models (with a not very clear density dependent NSA). For these nuclei ($\delta_{av.} \simeq \delta_0/2$) our estimate of the NSA is of 0.10 MeV. This is consistent with the values quoted in table 1 both *in sign and magnitude* although not big enough to completely remove the anomaly.

These results for the n-p mass difference within dense matter, suggest that in this case one should include, besides the $\rho - \omega$ mixing, other contributions which, although of minor importance in vacuum, seem to be relevant inside nuclei.

According to the vigor recently regained by the ρ - ω mixing effective interaction [19], in this work we have extended our previous analysis on the mass splitting of an isolated nucleon [13]. In the present article we have gone a step further, considering the effects of temperature and density on this CSB outcome.

For standard astrophysical temperatures, far from the deconfining phase, we have found a negligible temperature effect on the nucleon self-masses. On the other hand, the density

effects are significant and were shown to produce the expected trends to remove the NSA.

REFERENCES

- [1] S. Weinberg, Phys. Rev. **D9** (1974), 3357. L. Dolan and R. Jackiw, Phys. Rev. **D9** (1974), 3320.
- [2] J. Kogut et al., Phys. Rev. Lett. **48** (1982), 1140. L. McLerran and B. Svetitsky, Phys. Lett. **B98** (1981), 195. J. Polonyi et al., Phys. Rev. Lett. **53** (1984), 664.
- [3] N. Bilic, J. Cleymans and M. D. Scadron, Int. Jour. Mod. Phys. **A10** (1995) 1169; J. Cleymans, A. Kocic and M. D. Scadron, Phys. Rev. **C39** 323; D. Bailin, J. Cleymans and M. D. Scadron Phys. Rev. **C31** (1985) 164.
- [4] European Muon Collaboration, J. Aubert et al., Phys. Lett. **B123** (1983), 275.
- [5] L. S. Celenza, A. Rosenthal and C. M. Shakin, Phys. Rev. Lett. **53** (1984), 892.
- [6] R. Jaffe, F. Close, R. Roberts and G. Ross, Phys. Lett. **B134** (1984), 449.
- [7] J. A. Nolen, J. P. Schiffer, Annu. Rev. Nucl. Sci **19** (1969), 471.
- [8] L. N. Epele, H. Fanchiotti, C. A. Garcia Canal G. A. Gonzalez Sprinberg and R. Mendez Galain, Phys. Lett. **B277** (1992), 33; L. N. Epele, H. Fanchiotti, C. A. Garcia Canal and R. Mendez Galain, Phys. Rev. **D39** (1989) R1473.
- [9] T. Hatsuda, H. Hogaansen and M. Prakash, Phys. Rev. Lett. **66** (1991), 2851. E. Henley and G. Krein, Phys. Rev. Lett. **62** (1989), 2586.
- [10] G. Krein, D. P. Menezes and M. Nielsen, Phys. Lett. **B294** (1992), 7. L. A. Barreiro, A. P. Galeao and G. Krein, Phys. Lett. **B358** (1995) 7.
- [11] *Proceedings of Quark Matter '90*, Nucl. Phys. **A525** (1991), C. P. Singh, Phys. Rep. **236** (1993) 147.
- [12] K. Erkelenz, Phys. Rep. **13**, 191 (1974); K. Holinde, Phys. Rep. **68**, 121 (1981); R. Machleidt, K. Holinde and Ch. Elster, Phys. Rep. **149**, 1 (1987).

- [13] H. R. Christiansen, L. N. Epele, H. Fanchiotti and C. A. Garcia Canal, Phys. Lett. **B267**, 164 (1991).
- [14] S. Glashow, Phys. Rev. Lett. **7** (1961) 469; Y. Nambu and J. J. Sakurai, Phys. Rev. Lett. **8** (1962) 949.
- [15] P. C. Mc Namee, M. D. Scadron and S. A. Coon, Nucl. Phys. **A249**, 483 (1975); S. A. Coon, M. D. Scadron and P. C. Mc Namee, Nucl. Phys. **287**, 381 (1977).
- [16] S. Coon and R. Barret, Phys. Rev. **C36**, 2189 (1987).
- [17] P. Langacker, Phys. Rev. **D19** (1979) 2983; P. Langacker and H. Pagels, Phys. Rev. **D19** (1979) 2070.
- [18] G. A. Miller, B. M. K. Nefkens and I. Slaus, Phys. Rep. **194** (1990) 1.
- [19] M. J. Iqbal, X. Jin and D. B. Leinweber, *Rho-omega mixing via QCD sum rules with finite mesonic widths*, TRI-PP-95-47, nucl-th/9507026.
- [20] S. A. Coon and M. D. Scadron, Phys. Rev. **C51** (1995) 2923.
- [21] R. Friedrich and A. Reinhardt, *Rho-omega mixing and the pion e-m form factors in the NJL model*. UNITU-THEP-1/1995, hep-ph/9501333.
- [22] M. Cini, E. Ferrari and R. Gatto, Phys. Rev. **2**, 7 (1959).
- [23] J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).
- [24] E. M. Henley and G. A. Miller in *Mesons and Nuclei*, edited by M. Rho and D. H. Wilkinson, (North Holland, Amsterdam, 1979) pag. 433; A. G. Williams, A. W. Thomas and G. A. Miller, Phys. Rev. **C36**, 1956 (1987); M. Iqbal and J. Niskanen, Phys. Rev. **C38**, 2259 (1988); M. Beyer and A. G. Williams, Phys. Rev. **C38**, 779 (1988); M. Iqbal, J. Thaler and R. Woloshyn, Phys. Rev. **C36**, 2442 (1987); C. Cheung, E. M. Henley and G. A. Miller, Nucl. Phys. **A305**, 342 (1978); *ibid.* **348**, 365 (1980);

- [25] Y. Takahashi and H. Umezawa, Collect. Phen. **2** (1975) 55; H. Matsumoto, Fortschr. Phys. **25** (1977) 1.
- [26] A. J. Niemi, G. W. Semenoff, Ann. Phys. **152** (1984) 105; N. P. Landsman and Ch. G. van Weert, Phys. Rep. **145** (1987) 141.
- [27] A major difficulty arising in the well-known Matsubara formalism (ITF) is that one has to deal with Euclidean propagators and Green functions with imaginary time arguments. In principle, real time quantities can be obtained by analytic continuation to the real axis but in practice the proper procedure is not always self evident and may become a difficult task. Nevertheless, at the one loop level, it can be proved that the physical meaning of the different continuation procedures are the same and the mentioned real quantities coincide with those obtained in a real time formalism (See Landsman and van Weert referenced above).
- [28] K. Saito, T. Maruyama and K. Soutome Phys. Rev. **C40** (1989) 407.
- [29] R. Kubo in *Statistical Mechanics* North Holland Pub.Co. Amsterdam, 1965.
- [30] L. Landau, E. Lifchitz and L. Pitayevski, *Physique Statistique*, Editions Mir, Moscou, 1967 (see ch.5 and §27,45.)
- [31] A. L. Fetter, J. D. Wallecka in *Quantum Theory of Many-Particle Systems*, Mc Graw-Hill Pub.Co. 1971.
- [32] S. Shlomo, Rep. Progr. Phys. **41** (1978), 957.
- [33] We have used the vacuum values for masses and couplings. The deviation from these numbers, a higher order correction, is negligible at low temperatures and densities. See e.g. C. Adami and G. E. Brown, Phys. Rep. **234** (1993) 1; S. Gao, R-K. Su and P. K. N. Yu, Phys. Rev. **C49** (1994) 40, and references therein.
- [34] H. Sato, Nucl. Phys. **A269** (1976) 378.

- [35] J. Piekarewicz and A. G. Williams, Phys. Rev. **C47** (1993) R2462; H. B. O'Connell, B. C. Pierce, A. W. Thomas and A. G. Williams, Phys. Lett **B336** (1994) 1.
- [36] G. A. Miller and W. T. H. van Oers, *Charge Independence and Charge Symmetry*, nucl-th/9409013.
- [37] A. W. Thomas and K. Saito, *Charge Symmetry Violation in Nuclear Physics*, ADP-95-37/T191.

TABLES

Table I. Values of the anomaly NSA in MeV reported in Refs. [7,32,34,9]. A is the mass number of mirror nuclei.

A	Nolen-Schiffer NSA [7]	Shlomo NSA [32]	Sato NSA [34]	Hatsuda et al NSA [9]
O-N 15	-	0.16	0.29	0.53
F-O 17	0.31	0.31	0.19	0.29
Si-P 29	0.20	-	0.24	-
S-Cl 33	0.24	-	0.28	-
Ca-K 39	-	0.22	0.43	0.57
Sc-Ca 41	0.62	0.59	0.35	0.42

FIGURES

Fig. 1. Diagrams contributing to the second term in eq. 6. The broken line represents the $T - \delta$ correction to the nucleon propagator, *i.e.* : $S^{11,z}$ (see eq. 3).

Fig. 2. Neutron-proton mass difference as a function of the hadronic density δ in units of δ_0 , the nuclear matter density, at $T=0$.

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9602244v1>